DOI: 10.31276/VJSTE.63(2).39-44

Optimum sizing of members of truss structures using direct design and a self-adaptive mutation differential evolution

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Received 16 April 2020; revised 30 June 2020; accepted 15 July 2020

Abstract:

Direct design using nonlinear inelastic analysis has been recently enabled for structural design as this approach can directly predict the behaviour of a structure as a whole, which eliminates capacity checks for individual structural members. However, the use of direct design is often accompanied by excessive computational efforts, especially for complicated structural design problems such as optimization or reliability analysis. In this study, we introduce an efficient method for the sizing optimization of truss structures employing nonlinear inelastic analysis for the direct design of structures. The objective function is the total weight of the structure while the strength and serviceability constraints are evaluated with nonlinear inelastic analysis. To save computational cost, an improved differential evolution (DE) algorithm is employed. Compared to the conventional DE algorithm, the proposed method has two major improvements: (1) a self-adaptive mutation strategy based on the *p-best* method to enhance the balance between global and local searches and (2) use of the multi-comparison technique (MCT) to reduce redundant structural analyses. The numerical results of a 72-bar truss case study demonstrate that the performance of the proposed method has significant advantages over the traditional DE method.

Keywords: **differential evolution, direct design, nonlinear inelastic analysis, optimization, truss.**

Classification number: **2.3**

1. Introduction

Conventional approaches such as allowable stress design (ASD), load and resistance factor design (LRFD), and the direct design paradigm are presented in Fig. 1. In these conventional techniques, a two-step member-based method is used where elastic analysis is first employed to calculate the forces acting on the structural members and then the strength equations provided from the design codes are applied to the strength checks of each member. Within these methods, strength and stability interactions between the whole structure and its elements are not considered. Further, the compatibility between structural members and the whole system is not guaranteed and therefore cannot ensure that all members will maintain their design loads under the geometric configuration of the structure. In contrast, direct design using nonlinear inelastic analysis allows the direct capture of both material nonlinear and geometric inelastic behaviours of the structure [1-5]. Thereby, the capacity check for individual structural elements, which is a requirement of the old paradigms, is eliminated. However, compared to the old paradigms, direct design requires excessive computation effort especially for complicated structural design problems such as optimization or reliability analysis [6-18].

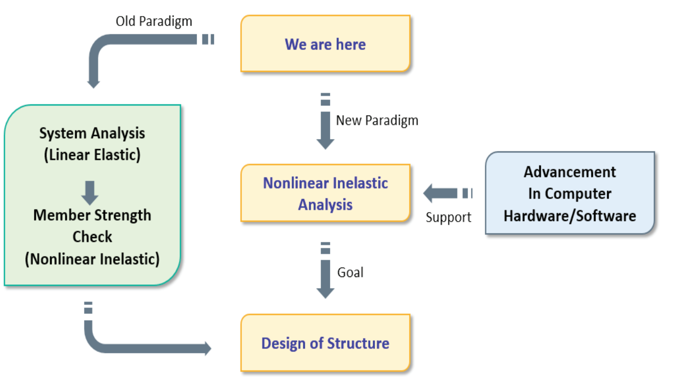

Fig. 1. Structural design concept [1].

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Sizing optimization of truss structures has been favoured recently as it leads to a substantial reduction in cost. In sizing optimization, the cross-sectional areas of the members are optimized such that the total weight of the structure is minimized while all design requirements are guaranteed. The sizing optimization of truss structures is often considered as a discrete engineering optimization problem, where the design variables (member cross-sectional areas) can take only discrete values. To evaluate the constraints that are related to the strength and the serviceability of the truss structure during the optimization procedure, direct design using nonlinear inelastic analysis has been preferred by many researchers because this approach can capture the nonlinear behaviours of the structure and yield more realistic and lighter optimum results [7, 9, 10, 12]. The aforementioned issues imply that a sizing optimization of truss structures using nonlinear inelastic analysis is highly nonlinear, non-convex, and multimodal. To solve this optimization problem, preference is given to meta-heuristic optimization algorithms, which implement searching techniques of a stochastic nature to select potential solutions in a given search space [7, 9, 10, 12]. However, the use of meta-heuristic optimization algorithms necessitates one to conduct a large number of structural analyses that render the optimization process very computationally demanding.

In this study, an efficient method for sizing optimization of steel trusses using nonlinear inelastic analysis is introduced. The objective function is the structural total weight. Nonlinear inelastic analysis is used to calculate the constraint functions regarding strength and serviceability. To reduce the computation efforts of the optimization process, a DE algorithm is employed with two major improvements as follows: (1) a self-adaptive mutation strategy that uses the *p-best* method to enhance the balance between global and local searches and (2) the MCT to reduce redundant structural analyses. A 72 bar-space truss is then studied to demonstrate the efficiency of the proposed method.

2. Formulation of sizing truss optimization problem

The objective function of the optimization is the total weight of a truss structure and the design variables are cross-sectional areas of the truss elements. The design constraints include strength and serviceability constraints. The problem is typically formulated as follows:

$$\begin{aligned}
&\textbf{Minimize} && W(\mathbf{Y})=\sum_{i=1}^{nm}\rho_i A_i L_i \\
&\textbf{Subjected to} && 1-\frac{R_j}{S_j}\le 0 \quad j=1,..,nstr \\
& && \frac{|d_{k,l}|}{|d_{k,l}^{u}|}-1\le 0 \quad k=1,..,nser;\ l=1,..,nn \\
& && A_i\subset \mathbf{S}_i
\end{aligned} \tag{1}$$

where ρ_i, A_i, and L_i are the material density, cross-sectional area, and length of the i^{th} element, respectively, S_i is the list of cross-sectional areas used for the i^{th} element, $\mathrm{Y}=(A_1, A_2,..., A_n)$ is the vector of design variables, R_i and S_j are the load-carrying capacity of the structure and the factored loading with the j^{th} strength load combination, respectively, $d_{k,l}$ and $d_{k,l}^{u}$ are the displacement of the node l and its allowable value with the k^{th} serviceability load combination, respectively, nm and nn are the numbers of truss elements and nodes, respectively, and $nstr$ and $nser$ are the numbers of strength and serviceability load combinations considered, respectively. In Eq. (1), $\frac{R_j}{S_j}$ can be designated as the ultimate load factor of the truss with the j^{th} strength load combination and is directly determined by using nonlinear inelastic analysis. The nodal displacements $d_{k,l}$ are also calculated by performing nonlinear elastic analysis. In this study, the practical advanced analysis program (PAAP) [2], which is efficient software for nonlinear inelastic analysis of steel structures like trusses and frames, is used to calculate the structural ultimate load factor and nodal displacements. A detailed implementation of PAAP for truss structures can be found in Refs [2, 19].

The objective function in Eq. (1) is transformed into unconstrained one as:

$$W_{uncstr}(\mathbf{Y})=\left(1+\sum_{j=1}^{nstr}\alpha_{str,j}\beta_{1,j}+\sum_{k=1}^{nser}\alpha_{disp,k}\beta_{2,k}\right)\times\left(\sum_{i=1}^{nm}\rho_i A_i L_i\right) \tag{2}$$

where

$$\begin{aligned}
\beta_{1,j} &= \max\left(1-\frac{R_j}{S_j},0\right);\ j=1,..,nstr \\
\beta_{2,k} &= \sum_{l=1}^{nn}\max\left(\frac{|d_{k,l}|}{|d_{k,l}^{u}|}-1,0\right);k=1,..,nser
\end{aligned} \tag{3}$$

in which $\alpha_{str,j}$ and $\alpha_{disp,k}$ are the penalty factors corresponding to the strength load combination j^{th} and serviceability load combination k^{th}, respectively.

3. Self-adaptive mutation differential evolution

The DE algorithm was developed by R. Storn, et al. (1997) [20] and has been considered as one of the most efficient methods. “DE/rand/1” and “DE/best/1”, two popular mutation strategies in DE, have opposite ways to balance the global and local searches of the optimization process. In “DE/rand/1”, the generation of the trial individual is based on an individual selected at random, so this strategy is good for global exploration and maintenance of the diversity of the population, but it has a low converge rate. On the contrary, in “DE/best/1” the trial individual is created using the current best individual of the population; hence, this strategy is effective in local search and increases the convergence speed but can easily get trapped into locally optimum results. Obviously, to balance the global and local searches of the optimization process a combination of both the above mutation strategies is an ideal approach. Particularly, at the early stage of the optimization process the population diverges making 'DE/rand/1' preferable, while "DE/best/1" is favoured at the late stage of the optimization process when the population starts to converge. In light of this, a mutation strategy developed using the *p-best* method is employed in this study. In this method, the trial individual is created by using a random member of the *p%* best individuals of the current population. The *p* value is calculated using a self-adaptive equation as follows:

$$p = \frac{1}{NP} + \left(1 - \frac{1}{NP}\right) \times \frac{DI_{(t)}}{DI_{(0)}} \qquad (4)$$

where

$$DI_{(t)} = \frac{1}{NP}\sum_{j=1}^{NP}\sqrt{\sum_{i=1}^{D}\left(\frac{x_{j,i} - \frac{1}{NP}\sum_{j=1}^{NP} x_{j,i}}{x_i^{UB} - x_i^{LB}}\right)^2} \qquad (5)$$

where D and NP are the number of design variables and individuals, respectively; $x_{j,i}$ is the i^{th} design variable of the j^{th} individual; x_i^{LB} and x_i^{UB} are the lower- and upper-bounds of $x_{j,i}$, respectively; and $DI_{(t)}$ is a diversity index of the population at the t^{th} generation. As presented in Eq. (5), $DI_{(t)}$ represents the individual distribution around the centre of the current population. From Eq. (4), the value of p is dependent on $DI_{(t)}$. Therefore, if $DI_{(t)}$ is large, which means that the individuals are still highly dispersed, a large p value is used and vice versa.

On the other hand, the strength and serviceability constraints are evaluated through the use of nonlinear analysis. The total structural analyses required are equal to the product of the total strength and serviceability load combinations under consideration, the population size, and the number of generations. For example, with 3 strength load combinations, 1 serviceability load combination, 20 individuals in the population, and 400 generations, the number of structural analyses is 32,000. Due to this fact, the optimization process takes an excessive amount of computational time. Note that, in the selection operator of the DE method, the trial individual will replace the target individual in the population if its objective function is smaller than that of the target individuals, otherwise it is neglected. Therefore, to reduce computation costs, the strength and serviceability constraints are evaluated step-by-step. After each step, the corresponding cumulative value of the trial individual's objective function is determined. Afterward, the program checks if it is higher than the target individual's objective function. If so, the trial individual is immediately ignored without evaluation of the remaining constraints. This method is so-called the multi-comparison technique (MCT) [9]. Further details of this method can be found in V.H. Truong, et al. (2018a) [9] research. Besides, the discrete variables are solved by rounding the continuous variables to the nearest value of the discrete variables.

4. Case study

In this section, a 72-bar truss, presented in Fig. 2, is optimized to show the efficiency of the proposed optimization method. The material properties are as follows: an elastic modulus of 68.95 (GPa), a yield strength of 172.375 (MPa), and a density of 2,767.99 (kg/m^3). The applied loads are a dead load (DL) of 100 kN, live load (LL) of 100 kN, and wind load (W) of 100 kN, which are applied to the structure as point loads as presented in Fig. 2. The cross-sectional areas of all 72 elements are classified into 16 design groups as presented in Table 1. A discrete list of the cross-sectional areas of the truss element including 42 values is also shown in Table 1. Three strength and one serviceability load combinations are considered such as (1.4DL), (1.2DL+1.6LL), (1.2 DL+0.5 LL+1.7W), and (1.0DL+0.5LL+0.7W), respectively. The drift constraint for the serviceability load combination is equal to *h/400* where *h* is the story height.

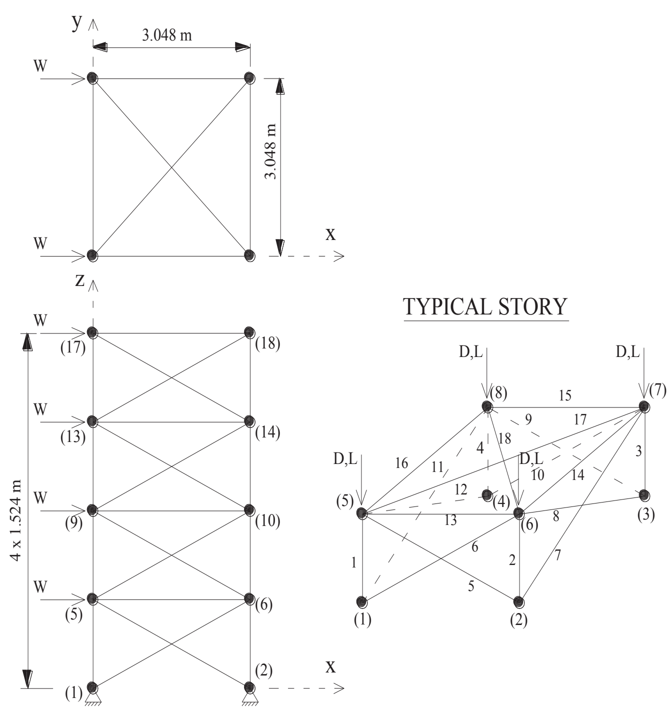


Fig. 2. 72-bar truss.

Table 1. Information of design groups of spatial 72-bar truss.

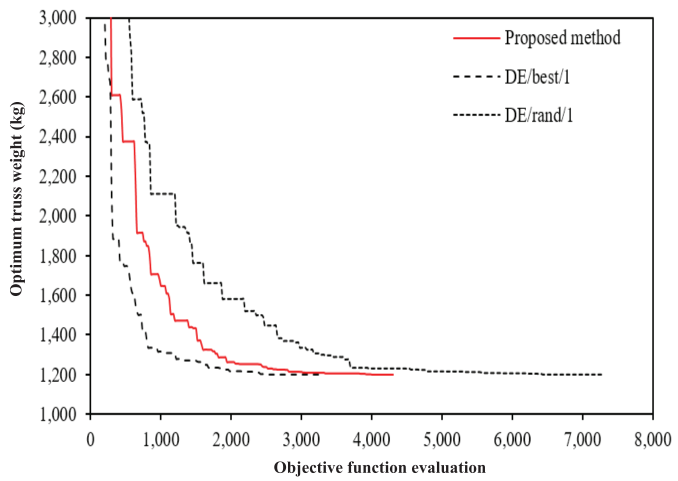
Element group number	Content
1	Element 1 to 4
2	Element 5 to 12
3	Element 13 to 16
4	Element 17 to 18
5	Element 19 to 22
6	Element 23 to 30
7	Element 31 to 34
8	Element 35 to 36
9	Element 37 to 40
10	Element 41 to 48
11	Element 49 to 52
12	Element 53 to 54
13	Element 55 to 58
14	Element 59 to 66
15	Element 67 to 70
16	Element 71 to 72
List for cross-sectional areas (in^2)	1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5

The traditional DE method with the two mutation operators 'DE/rand/1' and 'DE/best/1' is used for comparison with the proposed method. To save computation time, the MCT method is implemented into the traditional DE method. The parameters used for all considered methods are NP=20; D=16; maximum value of generations =400; scale factor F=0.7; and crossover factor CR=0.6. The termination of the optimization process is defined as (1) the number of generations reaches 400 or (2) the difference between the worst and the best individuals of the population is smaller than 0.0001.

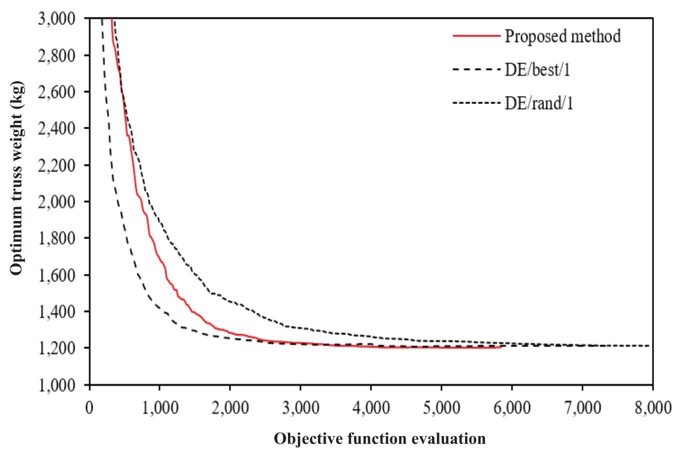
Table 2 presents the optimization results where each method is performed 10 times. All methods yield the same best optimum design with a total weight of 1,200.1 (kg). However, the proposed method has a greater stability than the other methods because it produces the lowest values of the unfavourable weight, the average weight, and standard deviation (std) of the optimum weights. The performance of the "DE/best/1" method is the worst with the highest value of the worst weight. This implies that this method was trapped in a local optimum. Regarding the "DE/rand/1" method, its performance converged more slowly than other methods as it required an average of 395 generations. Therefore, the optimum results of the "DE/rand/1" method are worse than the proposed method. Furthermore, Table 1 indicates that the proposed method requires only an average of 9,369 structural analyses for 227 generations. This means that it saves about 48.4% required structural analyses compared to using the traditional DE method (without using MCT). Also, Figs. 3A and 3B present the histories of the best and average scores during the optimization procedure, respectively. The proposed method converges more slowly than "DE/best/1" but more quickly than "DE/rand/1".

Table 2. Optimization results of spatial 72-bar truss.

Element group number	Proposed method (mm^2)	DE/best/1 (mm^2)	DE/rand/1 (mm^2)
1	9,999.980	9,999.980	9,999.980
2	2,238.705	2,238.705	2,238.705
3	1,045.159	1,045.159	1,045.159
4	1,045.159	1,045.159	1,045.159
5	7,419.340	7,419.340	7,419.340
6	2,290.318	2,290.318	2,290.318
7	1,045.159	1,045.159	1,045.159
8	1,045.159	1,045.159	1,045.159
9	3,703.218	3,703.218	3,703.218
10	1,690.319	1,690.319	1,690.319
11	1,045.159	1,045.159	1,045.159
12	1,045.159	1,045.159	1,045.159
13	1,045.159	1,045.159	1,045.159
14	1,535.481	1,535.481	1,535.481
15	1,045.159	1,045.159	1,045.159
16	1,045.159	1,045.159	1,045.159
Best weight (kg)	1,200.100	1,200.100	1,200.100
Worst weight (kg)	1,210.800	1,276.700	1,230.800
Average weight (kg)	1,203.782	1,212.638	1,207.927
Std. weight (kg)	4.292	20.572	9.969
Average objective function evaluations	9,369	9,720	12,844
Average number of generations	227	220	395



(A)



(B)

Fig. 3. Convergence histories of 72-bar truss. (A) The best optimum designs; **(B)** The average optimum designs.

5. Conclusions

An efficient method for the sizing optimization of truss structures was successfully developed. Direct design using nonlinear inelastic analysis was applied to the evaluation of constraints related to strength and serviceability. An improved differential evolution algorithm was developed using a self-adaptive mutation strategy to enhance the balance between global and local searches along with a multi-comparison technique to reduce redundant structural analyses. The traditional DE method with the two popular mutation operators "DE/best/1" and "DE/rand/1" were used to demonstrate the accuracy and efficiency of the proposed optimization method. The numerical results of a 72 bar-space truss prove that the proposed method's performance was better than that of the traditional DE method.

CRediT author statement

Manh-Hung Ha, Hoang-Anh Pham: Conceptualisation, Methodology, Software, Resources, Writing - Review and Editing, Validation.

ACKNOWLEDGEMENTS

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 107.01-2018.327.

COMPETING INTERESTS

The authors declare that there is no conflict of interest regarding the publication of this article.

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