The effect of the dielectric layer thickness on the negative permeability in metamaterials

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Abstract:
In this work, we investigate the influences of the dielectric layer on the magnetic resonance of the cut-wire pair structure (CWP). The interaction between the cut-wires is modeled through a LC circuit, based on which, the magnetic resonant frequency is calculated. Furthermore, the dependence of the resonant bandwidth on the structure parameters is also determined. By tuning the dielectric layer thickness, we obtained a noticeable broadening of the negative permeability regime of 17%, which represents the enhancement of the magnetic resonance. A good agreement between the theory, simulation, and practical experiment has been demonstrated. We believe that our results should be consequential with regard to the determination of the mechanism behind the wave-matter interaction in the GHz frequency regime.

***Keywords:** dielectric layer thickness, metamaterials, negative permeability broadening.*

***Classification number:** 2.1*

Introduction

In recent years, the revolution in science and technology with regard to seeking novel materials has gained tremendous popularity throughout the world. Metamaterials (MMs) have been one of the most prominent candidates in this regard due to their extraordinary properties. Numerous potential applications of MMs have been proposed and demonstrated, such as biological sensor [1], superlens [2], hi-low pass filtering [3], antennas [4], invisible cloaking [5], and wireless power transfer [6]. Most of these applications are based on the unique optical property of negative refractive index in MMs [7-9]. Whereas, the permeability and permittivity of MMs are simultaneously negative in a common frequency regime [10-13]. This unique property of MMs is known to be arbitrarily tuned in terms of the arrangement or design of their compositions. In fact, the negative permittivity region can be obtained on a wide scale through the periodic continuous-wire structure [14], but the negative permeability region is restricted due to the resonant conditions. Therefore, the realistic applications of MMs are limited by the narrow negative permeability bandwidth. From the large amount of efforts made to expand the negative permeability of MMs [15-17], the thickness of the dielectric layer has been concluded to be extremely significant. However, no systematic study on the influences of the dielectric layer thickness currently exists.

In this work, we report the effects of dielectric layer thickness on the negative permeability bandwidth of the conventional cut-wire pair (CWP) structure from 12 to 18 GHz. The negative permeability region is observed to be broadened as a result of the increased dielectric layer thickness. The phenomenon is interpreted theoretically by calculations on the LC circuit model and verified by simulations and experiments with considerable consistency.

Theoretical model

The CWP structure includes two metal patterns on two sides of the dielectric layer (Fig. 1A). Due to Zhou, et al.'s great work [18], the CWP structure can be modeled by an equivalent LC circuit (Fig. 1B) by considering the CWs as the inductors and the spaces between the ends of the CW as the capacitors. Furthermore, the resonant frequency can be theoretically predicted as shown below:

$$f_m = \frac{c}{\pi l \sqrt{2\varepsilon c_1}} \quad (1)$$

Since Eq. 1, the resonant frequency depends on the CW length and the dielectric constant of the middle layer. However, some experimental results

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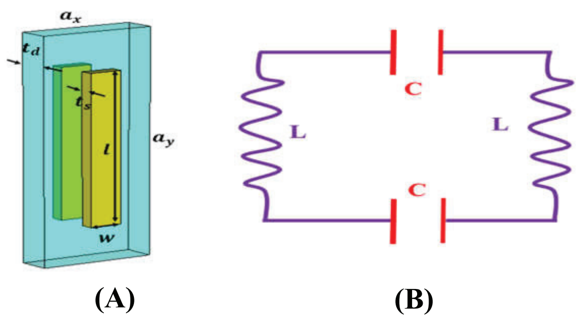

Fig. 1. (A) Unit cell and (B) the corresponding LC circuit of the CWP structure.

reveal that dielectric thickness also influences the resonant frequency [19, 20]. In our study, we propose a new equation to calculate the inductance and capacitance in the equivalent LC circuit, in which the dielectric thickness is considered according to the hybridization model:

$$L = \frac{\mu_0 l w}{2(w+t_s)^2}(t_d - 2t_s) \qquad C = \frac{\varepsilon\varepsilon_0 c_1 l w}{t_d} \qquad (2)$$

where: l is the length, w constitutes the width, and t_s forms the thickness of a CW; t_d represents the dielectric layer thickness and c_1 is a constant $0.2 \le c_1 \le 0.3$.

Hence, the resonant frequency of the equivalent LC circuit becomes.

$$f = \frac{c}{\pi l\sqrt{2\varepsilon c_1\left(1 - \frac{2t_s}{t_d}\right)/(1+\frac{t_s}{w})}} \qquad (3)$$

In Eq. 3, the resonant frequency is shown to depend on the the width of the CW and the dielectric layer thickness. It is worth noting that in the case where ($t_s \ll w$) and ($ts \ll t_d$), Eq. 3 assumes the same form as Eq. 1 of J. Zhou, et al. [18]. In other words, the mutual coupling effect between the CWs has not been included in the calculations of J. Zhou, et al. [18]. Therefore, the fractional bandwidth of the negative permeability can be expressed as follows:

$$\frac{\Delta f}{f_0} = \frac{1}{\sqrt{1 - F}} - 1 \qquad (4)$$

where: F is defined as:

$$F = \frac{l}{a_x a_y a_z}\frac{(w+t_s)^2}{w}\frac{t_d^2}{t_d - 2t_s} \qquad (5)$$

Since t_d is considered to fall in the range between 0.2 to 1.0 mm, F is always greater than 0 and smaller than 1. Hence, Eq. 4 is positive and identifiable. The relation between the dielectric layer thickness and the negative permeability bandwidth can be realized in Eq. 4 and Eq. 5 or more clearly in Fig. 2, where the evolution of $\Delta f/f_0$ according to t_d is presented.

Simulation and experiment

The proposed CWP structure is composed of a dielectric layer FR4 with the dielectric constant as 4.3 and copper patterns with t_s = 0.036 mm, l = 5.5 mm, w = 1 mm. The structure is periodic along the x and y axis; the lattice constant for each direction is a_x = 3.6 mm and a_y = 7.2 mm respectively. The thickness of the dielectric layer is tuned from 0.2 mm to 1.0 mm by a step of 0.2 mm. The samples are prepared on the standard printed circuit boards (PCB) through the application of the conventional photolithography method (Fig. 3). The simulations are operated on the simulation program CST [21], and the measurements are performed by the vector network analyzer system to obtain the scattering parameters.

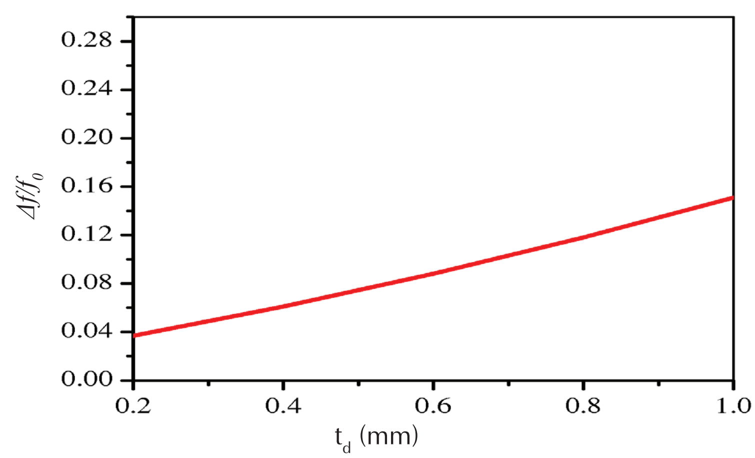

Fig. 2. The dependence of the negative permeability fractional bandwidth on the dielectric layer thickness presented by theoretical model.

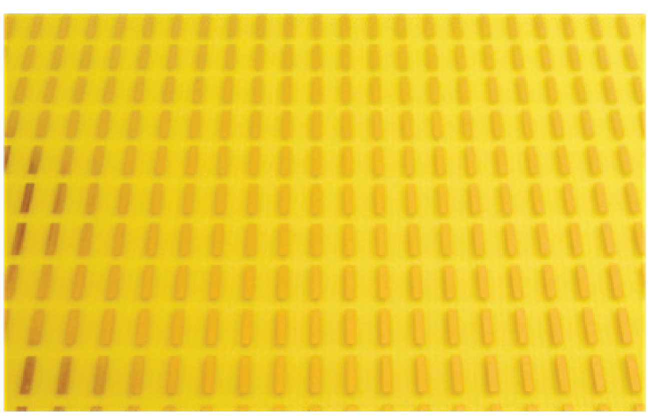

Fig. 3. Fabricated sample with the presented structure parameters.

Results and discussions

Figures 4A, 4B present the simulated and measured transmission spectra of the CWP structure at various dielectric layer thicknesses. As expected in Eq. 3, the resonant frequency shifted to the higher frequency regime when we increased the dielectric layer thickness from 0.4 to 1.0 mm.

The results with regard to magnetic resonant frequencies are listed in Table 1 with a comparison between the simulated, experimental, and theoretical calculations from this work and Zhou's work. The influence of the mutual coupling between the two cut-wires in a unit cell should be noticed when t_d is small, as discussed above. Moreover, the resonant peak is observed to demonstrate the blue shift in correspondence to the increase in the thickness of the dielectric layer. The rate of increase presents a good agreement between our calculations and data provided in Fig. 3 and a slightly higher correspondence with J. Zhou, et al.'s [18] work, confirming the improvement in the accuracy of our calculation in comparison to the reference.

In addition to the frequency shift along with the increasing thickness of the dielectric layer, the negative permeability regime is also expected to broaden. Fig. 5 presents numerical results of the transition spectra as a function of dielectric layer thickness from 0.1 to 1.0 mm. We observe that the abandon gaps of the transmission spectra are very narrow and shallow with the thin dielectric layer. Regardless, by increasing the dielectric layer thickness to 1.0 mm, the transmission gap is significantly widened and deepened. Since the magnetic resonance in the structure presents the electromagnetic waves propagating through the MM, the enlargement of the transmission regime may correspond to wider negative permeability.

Thanks to X. Chen, et al.'s work [22], from the scattering parameters obtained by simulation, the permeability spectra are extracted in Fig. 6 A

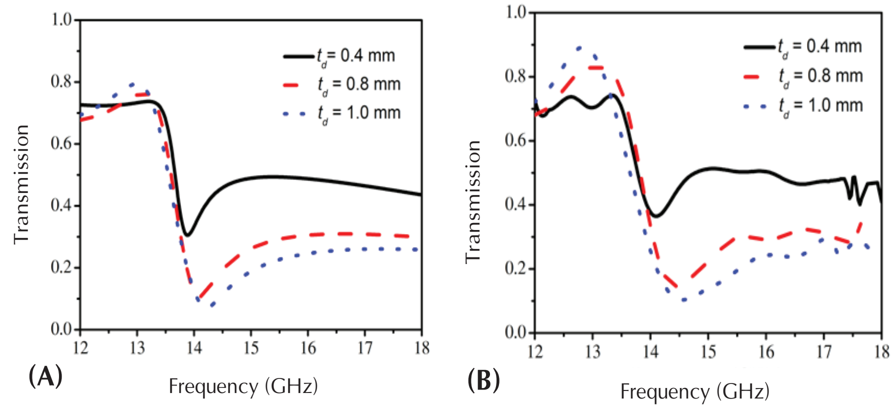

Fig. 4. The (A) simulated and (B) experimental transmission spectra of the CWP with t_d=0.4, 0.8, 1.0 mm.

Table 1. The magnetic resonant frequencies obtained in simulation, experiment and calculations in this work and reference [18].

f_m (GHz) / t_d (mm)	This work	Zhou's work	Simulation	experiment
0.4	14.02	12.41	13.968	14.076
0.8	14.18	13.01	14.172	14.139
1	14.33	13.28	14.268	14.51

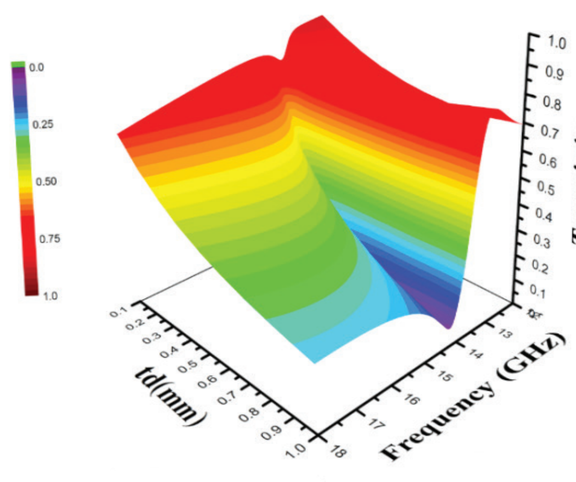

Fig. 5. The dependence of the transmission spectra on the dielectric layer thickness.

at t_d= 0.2, 0.4, 0.8, and 1.0 mm. A significant enhancement of the negative permeability can be observed. In fact, at 0.2 mm, the fractional bandwidth is only 4% (bandwidth of 0.55 GHz at the center frequency of 13.79 GHz) and at 1.0 mm, the frictional bandwidth is four times larger, up to 17% (bandwidth of 2.42 GHz at the center frequency of 14.27 GHz). This implies a good agreement with the simulation results. Our calculations following equation 2.8 exhibit an increase of frictional bandwidth from 3.6% at t_d = 0.2 mm to 14.4% at t_d = 1.0 mm as depicted in Fig. 6B. Our theoretical calculations also show good correspondence with the simulation results displayed in Fig. 6B.

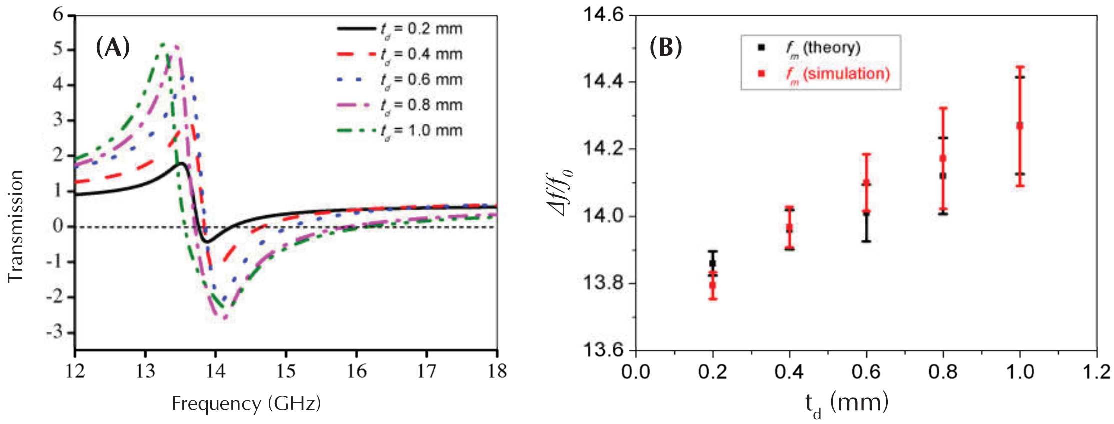

Fig. 6. (A) The dependence of the permeability on dielectric layer thickness at t_d = 0.2, 0.4, 0.8, 1 mm and (B) The fractional bandwidth of the negative permeability in theory and simulation.

Conclusions

We have investigated the influence of dielectric layer thickness on the negative permeability of the conventional CWP structure. The negative permeability region exhibits a blue shift and a broadening with increase in dielectric layer thickness. The LC circuit model was employed to interpret this behavior. A good agreement with simulations and experiments was obtained that confirms the validity of our analysis. The results will be useful in understanding the mechanism of wave-matter interaction in MMs in GHz frequency regime.

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